

**I. PART I. MULTIPLE CHOICE QUESTIONS (7,0 points)**

Write the correct answer (A, B, C or D) for each of the following questions in the correspondingly numbered space on your answer sheet.

**Question 1.** Given two non-zero vectors  $\vec{a}$  and  $\vec{b}$ . Which of the following statements is **false**?

- A. Two vectors  $\vec{a}$  and  $\vec{b}$  with the same direction are parallel.
- B. Two vectors  $\vec{a}$  and  $\vec{b}$  with opposite direction to another nonzero vector are parallel.
- C. Two vectors  $\vec{a}$  and  $-3\vec{a}$  have the same direction.
- D. Two vectors  $\vec{a}$  and  $k\vec{a}$  are parallel.

**Question 2.** In the  $Oxy$  coordinate plane, given  $A(1; -3)$  and  $B(-5; 4)$ . The coordinates of vector  $\overrightarrow{BA}$  are

- A.  $\overrightarrow{BA} = (6; 7)$
- B.  $\overrightarrow{BA} = (-4; 1)$ .
- C.  $\overrightarrow{BA} = (6; -7)$ .
- D.  $\overrightarrow{BA} = (-6; 7)$ .

**Question 3.** Which of the following two inequations are **not** equivalent?

- A.  $3x^2 + 1 \leq 2x - 1$  and  $3x^2 - 2x + 2 \leq 0$ .
- B.  $5x - 1 + \frac{1}{x-2} > \frac{1}{x-2}$  and  $5x - 1 > 0$ .
- C.  $-2x + 1 > 0$  and  $2x - 1 < 0$ .
- D.  $2x - 1 > 0$  and  $2x - 1 + \frac{1}{2x^2 + 1} > \frac{1}{2x^2 + 1}$ .

**Question 4.** In the  $Oxy$  coordinate plane, given  $\triangle ABC$  with  $A(-1; -4)$ ,  $B(6; 7)$  and  $C(-2; 9)$ . Let  $G$  be the centroid of  $\triangle ABC$ . The coordinates of  $G$  are

- A.  $G(1; 4)$ .
- B.  $G(-1; 4)$ .
- C.  $G(1; -4)$ .
- D.  $G(3; 12)$ .

**Question 5.** Given  $\triangle ABC$  with  $AB = 13$ ,  $BC = 2\sqrt{33}$ ,  $CA = 17$ . Compute the length of the median  $AM$  of  $\triangle ABC$ .

- A.  $AM = \sqrt{194}$ .
- B.  $AM = 2\sqrt{35}$ .
- C.  $AM = 15$ .
- D.  $AM = 14$ .

**Question 6.** Given equation  $(x^2 - x + 1)(x - 1)(x + 1) = 0$ . Which of the following equations is equivalent to the given equation?

- A.  $x + 1 = 0$ .
- B.  $x^2 - x + 1 = 0$ .
- C.  $x - 1 = 0$ .
- D.  $(x - 1)(x + 1) = 0$ .

**Question 7.** Given an isosceles right triangle  $ABC$  with sides  $AB = AC = 42\text{ cm}$ . Two medians  $BE$  and  $CF$  intersect at point  $G$ . The area of the triangle  $GEC$  is

- A.  $21\sqrt{7}\text{ cm}^2$ .
- B.  $174\text{ cm}^2$ .
- C.  $7\sqrt{21}\text{ cm}^2$ .
- D.  $147\text{ cm}^2$ .

**Question 8.** The negation of the proposition "Fourteen is a composite number" is

- A. Fourteen has four positive factors.
- B. Fourteen is not a composite number.
- C. Fourteen is a prime number.
- D. Fourteen has only two factors 1 and 14.

**Question 9.** In the  $Oxy$  coordinate plane, let  $A(-3; -5)$ ;  $B(2; 5)$ . Determine the slope of line  $AB$ .

- A. -5.
- B. 5.
- C. 2.
- D. -3.

**Question 10.** Given two sets  $X = \{A; 1; 2; 4; 6\}$ ,  $Y = \{3; 7; 4; \emptyset\}$ , the union of  $X$  and  $Y$  is

- A.  $\{A; 1; 2; 3; 4; 6; 7\}$ .
- B.  $\{1; 2; 3; 4; 5; 6; 7\}$ .
- C.  $\{1; 2; 3; 4; 6; 7\}$ .
- D.  $\{A; 1; 2; 3; 4; 6; 7; \emptyset\}$ .

**Question 11.** Given  $\triangle ABC$  with the sides  $AC = 3\sqrt{3}$ , side  $BC = 3\sqrt{2}$ ,  $A = 45^\circ$  and  $B > A + C$ . Compute the degree measure of  $\widehat{ABC}$ .

- A.  $\widehat{ABC} = 60^\circ$ .
- B.  $\widehat{ABC} = 150^\circ$ .
- C.  $\widehat{ABC} = 30^\circ$ .
- D.  $\widehat{ABC} = 120^\circ$ .

**Question 12.** Given two equations  $mx^2 - 2(m-1)x + m - 2 = 0$  and  $(m-2)x^2 - 3x + m^2 - 15 = 0$ . How many values of  $m$  which make these above equations equivalent?

- A. 3.                                      B. 1.                                      C. 0.                                      D. 2.

**Question 13.** In the  $Oxy$  coordinate plane, given  $\triangle ABC$ . Points  $M(-2; 3)$ ,  $N(4; -1)$ ,  $P(1; 1)$  are the mid-points of sides  $BC$ ,  $CA$  and  $AB$ , respectively. The coordinates of vertex  $A$  are

- A.  $A(10; 0)$ .                              B.  $A(-10; 0)$ .                              C.  $A(-7; 3)$ .                              D.  $A(7; -3)$ .

**Question 14.** In the  $Oxy$  coordinate plane, let Parabol  $(P): y = ax^2 + bx + 3$  and a point  $M(-1; 9)$  belongs to the graph of  $(P)$ . The symmetric axis of  $(P)$  has equation  $x = -2$ . Find the value of  $S = a + b$ .

- A. 16.                                      B. 6.                                      C. -10.                                      D. -6.

**Question 15.** The domain of the function  $y = \frac{2}{\sqrt{6-2x}}$  is

- A.  $D = (-\infty; 3]$ .                              B.  $D = (3; +\infty)$ .                              C.  $D = (-\infty; 3)$ .                              D.  $D = \mathbb{R} \setminus \{3\}$ .

**Question 16.** A ball is thrown straight up from 60 meters above the ground with a velocity of 20 meters per second (20 m/s). The height of the ball at second  $t$  after throwing can be computed by the quadratic function  $s(t) = -5t^2 + 20t + 60$ , where  $s(t)$  is in meters. After how many seconds does the ball hit the ground?

- A.  $t = 6$ .                                      B.  $t = 1$ .                                      C.  $t = 2$ .                                      D.  $t = 4$ .

**Question 17.** Given three distinct points  $A$ ,  $B$  and  $C$ . Which of the following statements is **true**?

- A.  $\overrightarrow{AB} + \overrightarrow{CA} = -\overrightarrow{BC}$ .                              B.  $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{CB}$ .                              C.  $\overrightarrow{CA} - \overrightarrow{AB} = \overrightarrow{CB}$ .                              D.  $\overrightarrow{BA} - \overrightarrow{BC} = \overrightarrow{AC}$ .

**Question 18.** Given a right triangle  $ABC$  at  $A$ . Which of the following statements is **false**?

- A.  $\overrightarrow{AB} \cdot \overrightarrow{AC} < \overrightarrow{BA} \cdot \overrightarrow{BC}$ .                              B.  $\overrightarrow{AB} \cdot \overrightarrow{BC} < \overrightarrow{CA} \cdot \overrightarrow{CB}$ .                              C.  $\overrightarrow{AC} \cdot \overrightarrow{BC} < \overrightarrow{BC} \cdot \overrightarrow{AB}$ .                              D.  $\overrightarrow{AC} \cdot \overrightarrow{CB} < \overrightarrow{AC} \cdot \overrightarrow{BC}$ .

**Question 19.** Find all parameters  $m$  such that equation  $x^2 + (m-1)x + m^2 - 1 = 0$  has two distinct roots and these roots have the same sign.

- A.  $\frac{-5}{3} < m < -1$ .                              B.  $-1 < m < 1$ .                              C.  $1 < m < \frac{5}{3}$ .                              D.  $m < -1$  or  $m > 1$ .

**Question 20.** Let  $a, b, c$  be real numbers and  $a + 2021c > b + 2021c$ . Which of the following statements is **true**?

- A.  $-2020a > -2020b$ .                              B.  $a^2 > b^2$ .                              C.  $\frac{1}{a} > \frac{1}{b}$ .                              D.  $2021a > 2021b$ .

**Question 21.** Given the fact that the system of equations  $\begin{cases} x^3(2+3y)=8 \\ (y^3-2)x=6 \end{cases}$  has exactly two distinct

solutions  $(x_1, y_1); (x_2, y_2)$ . The value of  $S = x_1^4 + y_1^4 + x_2^4 + y_2^4$  is

- A. 40.                                      B. 28.                                      C. 36.                                      D. 34.

**Question 22.** Find all values of  $m$  such that function  $y = (m-1)x + 2021$  is decreasing on its domain.

- A.  $m > 1$ .                                      B.  $m < 1$ .                                      C.  $m \geq 1$ .                                      D.  $m \leq 1$ .

**Question 23.** Given rectangle  $ABCD$  with  $AD = 2$ . Suppose that  $E$  is the point which lies on the side  $AB$  such that  $AE = 2BE$  and  $\sin \widehat{BDE} = \frac{1}{5}$ . Compute the length of the segment  $AB$ .

- A.  $AB = 2\sqrt{2}$ .                              B.  $AB = \sqrt{6}$ .                              C.  $AB = \sqrt{3}$ .                              D.  $AB = 3\sqrt{3}$ .

**Question 24.** In the  $Oxy$  coordinate plane, let  $M$  be the vertex of Parabol  $y = ax^2 + bx + c$  ( $a \neq 0$ ). The coordinates of  $M$  are

- A.  $\left(\frac{-b}{2a}; \frac{4ac-b^2}{4a}\right)$ .                              B.  $\left(\frac{-b}{4a}; \frac{4ac-b^2}{4a}\right)$ .                              C.  $\left(\frac{b}{2a}; \frac{4ac-b^2}{4a}\right)$ .                              D.  $\left(\frac{-b}{2a}; \frac{b^2-4ac}{4a}\right)$ .

**Question 25.** Given a right triangle  $ABC$  at  $B$  with  $AB = 2a$ ,  $AC = 5a$ . Compute the dot product  $\overrightarrow{AB} \cdot \overrightarrow{CA}$ .

- A.  $-4a^2$                       B.  $-5a^2$                       C.  $5a^2$                       D.  $4a^2$ .

**Question 26.** Among the following propositions, whose inverse proposition is **true**?

- A. If a quadrilateral is an isosceles trapezoid then its two diagonals have the same length.  
B. If  $n$  is a natural number then  $n$  is a real number.  
C. If two triangles are congruent then their corresponding angles are equal.  
D. If a triangle is not regular then it has at least one interior angle less than 60 degrees.

**Question 27.** Which of the following sentences is **not** a proposition?

- A. What a nice day!                      B. If " $3 + x = 4$ " then " $x = 1$ ".  
C. If " $1 + 2 = 7$ " then " $7$  is an odd number".                      D. Five divides twenty.

**Question 28.** In the  $Oxy$  coordinate plane, given  $A(2; -6)$ . Let  $B$  be the point which is symmetric to point  $A$  with respect to the origin  $O$ . Find the coordinates of point  $C$  satisfying that its horizontal coordinate equals  $-4$  and  $\triangle ABC$  has the right angle at  $C$ .

- A.  $C(2\sqrt{6}; -4)$  or  $C(-2\sqrt{6}; -4)$ .                      B.  $C(-4; -2\sqrt{6})$  or  $C(-4; 2\sqrt{6})$ .  
C.  $C(24; -4)$  or  $C(-24; -4)$ .                      D.  $C(-4; 24)$  or  $C(-4; -24)$ .

**Question 29.** In the  $Oxy$  coordinate plane, given Parabol  $(P): y = x^2 - 5x + 2m$ . Let  $S$  be the set of all values of  $m$  such that the Parabol  $(P)$  cuts  $Ox$  at two distinct points  $A, B$  satisfying  $OA = 4OB$ . Determine the sum of all elements of  $S$ .

- A.  $\frac{2}{9}$ .                      B.  $\frac{-32}{9}$ .                      C.  $\frac{-16}{9}$ .                      D. 2.

**Question 30.** Given  $\triangle ABC$ . Let  $M$  and  $N$  be the mid-points of sides  $AB$  and  $AC$ , respectively. Find the scalars  $m$  and  $n$  such that  $\overrightarrow{NM} = m\overrightarrow{AB} - n\overrightarrow{AC}$ .

- A.  $m = \frac{1}{2}, n = \frac{1}{2}$ .                      B.  $m = \frac{1}{2}, n = -\frac{1}{2}$ .                      C.  $m = -\frac{1}{2}, n = -\frac{1}{2}$ .                      D.  $m = -\frac{1}{2}, n = \frac{1}{2}$ .

**Question 31.** Given an isosceles triangle  $ABC$  with the right angle  $A$ , inscribed in a circle with center  $O$  and radius  $R$ . Let  $r$  be the radius of the incircle of triangle  $ABC$ . The ratio of  $R$  to  $r$  is

- A.  $\frac{R}{r} = \frac{\sqrt{2}-1}{2}$ .                      B.  $\frac{R}{r} = \frac{2+\sqrt{2}}{2}$ .                      C.  $\frac{R}{r} = 1 + \sqrt{2}$ .                      D.  $\frac{R}{r} = \frac{1+\sqrt{2}}{2}$ .

**Question 32.** Given  $A = \{1; 2; 3; 4\}$ . How many subsets does the set  $A$  have?

- A. 14.                      B. 15.                      C. 16.                      D. 18.

**Question 33.** In the  $Oxy$  coordinate plane, given two vectors  $\vec{a} = (6; -4)$  and  $\vec{b} = (-10; -2)$ . Compute the angle between two vectors  $\vec{a}$  and  $\vec{b}$ .

- A.  $45^\circ$ .                      B.  $60^\circ$ .                      C.  $120^\circ$ .                      D.  $135^\circ$ .

**Question 34.** A man travels from city  $X$  to city  $Y$  by train, then returns to city  $Y$  by his car. Given that the distance between these two cities is 200 km and the average speed of his car is 10 km/h faster than the train's average speed. His journey takes 9 hours, find the sum of average speeds of the train and his car.

- A. 60.                      B. 90.                      C. 80.                      D. 100.

**Question 35.** Let  $a, b, c$  be three positive real numbers satisfying  $a + b + c = 3$ . Determine the maximum value of  $T = \sqrt{ab} + \sqrt{bc} + \sqrt{ca}$ .

- A. 2.                      B. 4.                      C. 3.                      D. 6.

## II. PART II. PROBLEM SOLVING (3,0 points)

Write the solutions to the following problems in the provided space on your answer sheet.

### Problem 1. (1,0 point)

To measure the height of the Cham temple tower Po Klong Garai in Ninh Thuan province (Figure 1), two points  $A$  and  $B$  which are chosen on the ground with the length  $AB = 16m$  and the bottom  $C$  of the tower are collinear (Figure 2). Two total stations whose tripods have a height  $h = 1,6m$  are put at point  $A$  and point  $B$ . Let  $D$  be the top of the tower and two points  $A_1, B_1$  be collinear to  $C_1$  on height  $CD$  of the tower. The measurements are  $\widehat{DA_1C_1} = 54^\circ$  and  $\widehat{DB_1C_1} = 32^\circ$ . Calculate the height  $CD$  of the tower then round the result to 3 decimal places.



Figure 1

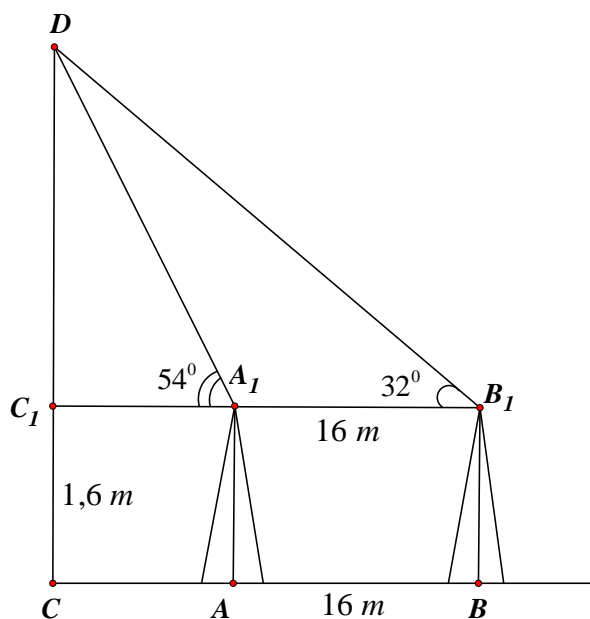


Figure 2

### Problem 2 (1,0 point).

Let  $f(x) = ax^3 + bx^2 + cx + d$  be a cubic function with  $f(0) = k$ ,  $f(1) = 2k$ ,  $f(-1) = 3k$ , where  $k$  is a given constant. What is the value of  $f(2) + f(-2)$ ?

### Problem 3 (1,0 point).

The sum of 2025 consecutive positive integers is a perfect square. Find the minimum value of the largest of these integers?

----- THE END -----

Student's full name: ..... Student's ID: .....

First observer's name and signature: ..... Second observer's name and signature: .....

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Write the correct answer (A, B, C or D) for each of the following questions in the correspondingly numbered space on your answer sheet.

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- A. -5.
- B. 5.
- C. 2.
- D. -3.

**Question 10.** Given two sets  $X = \{A; 1; 2; 4; 6\}$ ,  $Y = \{3; 7; 4; \emptyset\}$ , the union of  $X$  and  $Y$  is

- A.  $\{A; 1; 2; 3; 4; 6; 7\}$ .
- B.  $\{1; 2; 3; 4; 5; 6; 7\}$ .
- C.  $\{1; 2; 3; 4; 6; 7\}$ .
- D.  $\{A; 1; 2; 3; 4; 6; 7; \emptyset\}$ .

**Question 11.** Given  $\triangle ABC$  with the sides  $AC = 3\sqrt{3}$ , side  $BC = 3\sqrt{2}$ ,  $A = 45^\circ$  and  $B > A + C$ . Compute the degree measure of  $\widehat{ABC}$ .

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- A. 3.                                      B. 1.                                      C. 0.                                      D. 2.

**Question 13.** In the  $Oxy$  coordinate plane, given  $\triangle ABC$ . Points  $M(-2; 3)$ ,  $N(4; -1)$ ,  $P(1; 1)$  are the mid-points of sides  $BC$ ,  $CA$  and  $AB$ , respectively. The coordinates of vertex  $A$  are

- A.  $A(10; 0)$ .                              B.  $A(-10; 0)$ .                              C.  $A(-7; 3)$ .                              D.  $A(7; -3)$ .

**Question 14.** In the  $Oxy$  coordinate plane, let Parabol  $(P): y = ax^2 + bx + 3$  and a point  $M(-1; 9)$  belongs to the graph of  $(P)$ . The symmetric axis of  $(P)$  has equation  $x = -2$ . Find the value of  $S = a + b$ .

- A. 16.                                      B. 6.                                      C. -10.                                      D. -6.

**Question 15.** The domain of the function  $y = \frac{2}{\sqrt{6-2x}}$  is

- A.  $D = (-\infty; 3]$ .                              B.  $D = (3; +\infty)$ .                              C.  $D = (-\infty; 3)$ .                              D.  $D = \mathbb{R} \setminus \{3\}$ .

**Question 16.** A ball is thrown straight up from 60 meters above the ground with a velocity of 20 meters per second (20 m/s). The height of the ball at second  $t$  after throwing can be computed by the quadratic function  $s(t) = -5t^2 + 20t + 60$ , where  $s(t)$  is in meters. After how many seconds does the ball hit the ground?

- A.  $t = 6$ .                                      B.  $t = 1$ .                                      C.  $t = 2$ .                                      D.  $t = 4$ .

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- A.  $\overrightarrow{AB} + \overrightarrow{CA} = -\overrightarrow{BC}$ .                              B.  $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{CB}$ .                              C.  $\overrightarrow{CA} - \overrightarrow{AB} = \overrightarrow{CB}$ .                              D.  $\overrightarrow{BA} - \overrightarrow{BC} = \overrightarrow{AC}$ .

**Question 18.** Given a right triangle  $ABC$  at  $A$ . Which of the following statements is **false**?

- A.  $\overrightarrow{AB} \cdot \overrightarrow{AC} < \overrightarrow{BA} \cdot \overrightarrow{BC}$ .                              B.  $\overrightarrow{AB} \cdot \overrightarrow{BC} < \overrightarrow{CA} \cdot \overrightarrow{CB}$ .                              C.  $\overrightarrow{AC} \cdot \overrightarrow{BC} < \overrightarrow{BC} \cdot \overrightarrow{AB}$ .                              D.  $\overrightarrow{AC} \cdot \overrightarrow{CB} < \overrightarrow{AC} \cdot \overrightarrow{BC}$ .

**Question 19.** Find all parameters  $m$  such that equation  $x^2 + (m-1)x + m^2 - 1 = 0$  has two distinct roots and these roots have the same sign.

- A.  $\frac{-5}{3} < m < -1$ .                              B.  $-1 < m < 1$ .                              C.  $1 < m < \frac{5}{3}$ .                              D.  $m < -1$  or  $m > 1$ .

**Question 20.** Let  $a, b, c$  be real numbers and  $a + 2021c > b + 2021c$ . Which of the following statements is **true**?

- A.  $-2020a > -2020b$ .                              B.  $a^2 > b^2$ .                              C.  $\frac{1}{a} > \frac{1}{b}$ .                              D.  $2021a > 2021b$ .

**Question 21.** Given the fact that the system of equations  $\begin{cases} x^3(2+3y)=8 \\ (y^3-2)x=6 \end{cases}$  has exactly two distinct

solutions  $(x_1, y_1); (x_2, y_2)$ . The value of  $S = x_1^4 + y_1^4 + x_2^4 + y_2^4$  is

- A. 40.                                      B. 28.                                      C. 36.                                      D. 34.

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- A.  $m > 1$ .                                      B.  $m < 1$ .                                      C.  $m \geq 1$ .                                      D.  $m \leq 1$ .

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- A.  $AB = 2\sqrt{2}$ .                                      B.  $AB = \sqrt{6}$ .                                      C.  $AB = \sqrt{3}$ .                                      D.  $AB = 3\sqrt{3}$ .

**Question 24.** In the  $Oxy$  coordinate plane, let  $M$  be the vertex of Parabol  $y = ax^2 + bx + c$  ( $a \neq 0$ ). The coordinates of  $M$  are

- A.  $\left(\frac{-b}{2a}; \frac{4ac-b^2}{4a}\right)$ .                              B.  $\left(\frac{-b}{4a}; \frac{4ac-b^2}{4a}\right)$ .                              C.  $\left(\frac{b}{2a}; \frac{4ac-b^2}{4a}\right)$ .                              D.  $\left(\frac{-b}{2a}; \frac{b^2-4ac}{4a}\right)$ .

**Question 25.** Given a right triangle  $ABC$  at  $B$  with  $AB = 2a$ ,  $AC = 5a$ . Compute the dot product  $\overrightarrow{AB} \cdot \overrightarrow{CA}$ .

- A.  $-4a^2$                       B.  $-5a^2$                       C.  $5a^2$                       D.  $4a^2$ .

**Question 26.** Among the following propositions, whose inverse proposition is **true**?

- A. If a quadrilateral is an isosceles trapezoid then its two diagonals have the same length.  
B. If  $n$  is a natural number then  $n$  is a real number.  
C. If two triangles are congruent then their corresponding angles are equal.  
D. If a triangle is not regular then it has at least one interior angle less than 60 degrees.

**Question 27.** Which of the following sentences is **not** a proposition?

- A. What a nice day!                      B. If " $3 + x = 4$ " then " $x = 1$ ".  
C. If " $1 + 2 = 7$ " then " $7$  is an odd number".                      D. Five divides twenty.

**Question 28.** In the  $Oxy$  coordinate plane, given  $A(2; -6)$ . Let  $B$  be the point which is symmetric to point  $A$  with respect to the origin  $O$ . Find the coordinates of point  $C$  satisfying that its horizontal coordinate equals  $-4$  and  $\triangle ABC$  has the right angle at  $C$ .

- A.  $C(2\sqrt{6}; -4)$  or  $C(-2\sqrt{6}; -4)$ .                      B.  $C(-4; -2\sqrt{6})$  or  $C(-4; 2\sqrt{6})$ .  
C.  $C(24; -4)$  or  $C(-24; -4)$ .                      D.  $C(-4; 24)$  or  $C(-4; -24)$ .

**Question 29.** In the  $Oxy$  coordinate plane, given Parabol  $(P): y = x^2 - 5x + 2m$ . Let  $S$  be the set of all values of  $m$  such that the Parabol  $(P)$  cuts  $Ox$  at two distinct points  $A, B$  satisfying  $OA = 4OB$ . Determine the sum of all elements of  $S$ .

- A.  $\frac{2}{9}$ .                      B.  $\frac{-32}{9}$ .                      C.  $\frac{-16}{9}$ .                      D. 2.

**Question 30.** Given  $\triangle ABC$ . Let  $M$  and  $N$  be the mid-points of sides  $AB$  and  $AC$ , respectively. Find the scalars  $m$  and  $n$  such that  $\overrightarrow{NM} = m\overrightarrow{AB} - n\overrightarrow{AC}$ .

- A.  $m = \frac{1}{2}, n = \frac{1}{2}$ .                      B.  $m = \frac{1}{2}, n = -\frac{1}{2}$ .                      C.  $m = -\frac{1}{2}, n = -\frac{1}{2}$ .                      D.  $m = -\frac{1}{2}, n = \frac{1}{2}$ .

**Question 31.** Given an isosceles triangle  $ABC$  with the right angle  $A$ , inscribed in a circle with center  $O$  and radius  $R$ . Let  $r$  be the radius of the incircle of triangle  $ABC$ . The ratio of  $R$  to  $r$  is

- A.  $\frac{R}{r} = \frac{\sqrt{2}-1}{2}$ .                      B.  $\frac{R}{r} = \frac{2+\sqrt{2}}{2}$ .                      C.  $\frac{R}{r} = 1 + \sqrt{2}$ .                      D.  $\frac{R}{r} = \frac{1+\sqrt{2}}{2}$ .

**Question 32.** Given  $A = \{1; 2; 3; 4\}$ . How many subsets does the set  $A$  have?

- A. 14.                      B. 15.                      C. 16.                      D. 18.

**Question 33.** In the  $Oxy$  coordinate plane, given two vectors  $\vec{a} = (6; -4)$  and  $\vec{b} = (-10, -2)$ . Compute the angle between two vectors  $\vec{a}$  and  $\vec{b}$ .

- A.  $45^\circ$ .                      B.  $60^\circ$ .                      C.  $120^\circ$ .                      D.  $135^\circ$ .

**Question 34.** A man travels from city  $X$  to city  $Y$  by train, then returns to city  $Y$  by his car. Given that the distance between these two cities is 200 km and the average speed of his car is 10 km/h faster than the train's average speed. His journey takes 9 hours, find the sum of average speeds of the train and his car.

- A. 60.                      B. 90.                      C. 80.                      D. 100.

**Question 35.** Let  $a, b, c$  be three positive real numbers satisfying  $a + b + c = 3$ . Determine the maximum value of  $T = \sqrt{ab} + \sqrt{bc} + \sqrt{ca}$ .

- A. 2.                      B. 4.                      C. 3.                      D. 6.

## II. PART II. PROBLEM SOLVING (3,0 points)

Write the solutions to the following problems in the provided space on your answer sheet.

### Problem 1. (1,0 point)

To measure the height of the Cham temple tower Po Klong Garai in Ninh Thuan province (Figure 1), two points  $A$  and  $B$  which are chosen on the ground with the length  $AB = 16m$  and the bottom  $C$  of the tower are collinear (Figure 2). Two total stations whose tripods have a height  $h = 1,6m$  are put at point  $A$  and point  $B$ . Let  $D$  be the top of the tower and two points  $A_1, B_1$  be collinear to  $C_1$  on height  $CD$  of the tower. The measurements are  $\widehat{DA_1C_1} = 54^\circ$  and  $\widehat{DB_1C_1} = 32^\circ$ . Calculate the height  $CD$  of the tower then round the result to 3 decimal places.



Figure 1

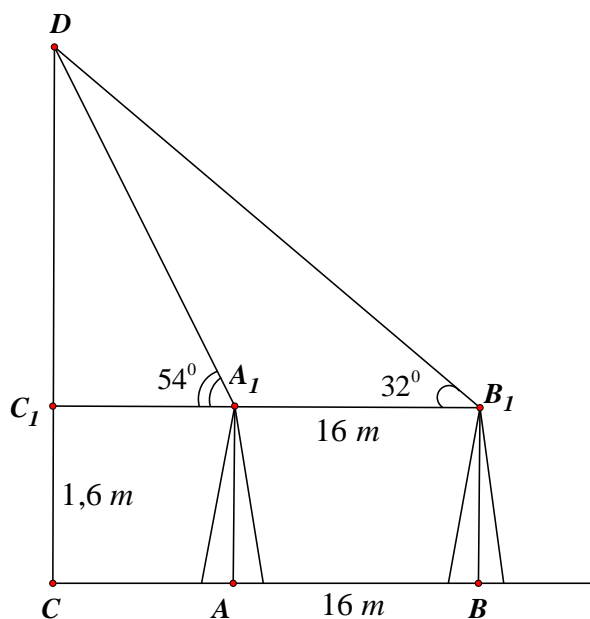


Figure 2

### Problem 2 (1,0 point).

Let  $f(x) = ax^3 + bx^2 + cx + d$  be a cubic function with  $f(0) = k$ ,  $f(1) = 2k$ ,  $f(-1) = 3k$ , where  $k$  is a given constant. What is the value of  $f(2) + f(-2)$ ?

### Problem 3 (1,0 point).

The sum of 2025 consecutive positive integers is a perfect square. Find the minimum value of the largest of these integers?

----- THE END -----

Student's full name: ..... Student's ID: .....

First observer's name and signature: ..... Second observer's name and signature: .....